Applications of Poromechanics to Energy Engineeering

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Modeling Thoughts...

« Toute loi physique, étant une loi approchée, est à la merci d'un progrès qui, en augmentant la précision des expériences, rendra insuffisant le degré d'approximation que comporte cette loi ; elle est essentiellement provisoire. L'appréciation de sa valeur varie d'un physicien à l'autre, au gré des moyens d'observation dont ils disposent et de l'exactitude que réclament leurs recherches ; elle est essentiellement relative. »

"Every law of Physics, being approximated, is at the mercy of progress, which, by increasing the accuracy of experiments, will make the degree of approximation of this law insufficient ; [every law of Physics] is essentially temporary. The appreciation of its value varies from one physicist to the other, depending on the means of observation that they have, and on the exactness required by their research work ; [every law of Physics] is essentially relative."

Pierre Duhem, La Théorie Physique. Son Objet - Sa Structure.
Chap. 5, § 3, Revue de Philosophie, 1906-1914.

1. **Thermodynamic Framework of Poromechanics**
2. **Performance Assessment of Heat Exchanger Piles**
3. **Modeling Damage in Porous Media**
4. **Study of the EDZ around Nuclear Waste Disposals**
5. **Prediction of Permeability in Fractured Rock**

Thermodynamics of Open Systems [Coussy, 2004]

- First Law of Thermodynamics (porous solid filled with a non reactive fluid mixture)

$$\dot{\mathbb{K}} + \dot{\mathbb{E}}_{int} = \dot{\mathbb{E}}_{tot} = \mathbb{P}_{mec} + \dot{\mathbb{E}}_{chem} + \dot{\mathbb{Q}}$$

$$\mathbb{P}_{mec} = \mathbb{P}_{defo} + \dot{\mathbb{K}} \Rightarrow \dot{\mathbb{E}}_{int} = \mathbb{P}_{defo} + \sum_j \mu_j \dot{N}_j + \dot{\mathbb{Q}}$$

$$\Psi = \mathbb{E}_{int} - TS \Rightarrow \dot{\Psi} + T\dot{S} + S\dot{T} = \mathbb{P}_{defo} + \sum_j \mu_j \dot{N}_j + \dot{\mathbb{Q}}$$

- Second Law of Thermodynamics

$$\dot{S} \geqslant \frac{\dot{\mathbb{Q}}}{T}$$

- Inequality of Clausius-Duhem

$$\Phi = \mathbb{P}_{defo} + \sum_j \mu_j \dot{N}_j - S\dot{T} - \dot{\Psi} \geqslant 0$$

Thermodynamic potentials depend on state variables and internal variables.

State Variables for Non-Isothermal Unsaturated Porous Media

2 miscible pure fluids (liquid, gas), small deformation, absence of body forces.

Inequality of Clausius-Duhem [Coussy, 2004]

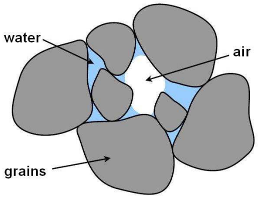
$$\Phi = \Phi_s + \Phi_l + \Phi_g + \Phi_T \geqslant 0$$

$$\Phi_s = \boldsymbol{\sigma} : \frac{d\epsilon}{dt} + p_l \frac{d^s \phi_l}{dt} + p_g \frac{d^s \phi_g}{dt} - S_s \frac{dT}{dt} - \frac{d^s \Psi_s}{dt}$$

$$\Phi_l = [-\boldsymbol{\nabla}_X p_l] \cdot \phi_l \mathbf{V_l^r}$$

$$\Phi_g = [-\boldsymbol{\nabla}_X p_g] \cdot \phi_g \mathbf{V_g^r}$$

$$\Phi_T = -\frac{\mathbf{Q}}{T} \cdot \nabla_X T$$



"deformation"	"stress"
$\boldsymbol{\epsilon^e}$	$\boldsymbol{\sigma}$
ϕ_l^e	p_l
ϕ_g^e	p_g
T	S_s

Thermodynamic Conjugation Relationships

The free energy of the solid skeleton is sought in the form :

$$\Psi_s = \Psi_s(\epsilon^e, \phi_l^e, \phi_g^e, T; \boldsymbol{\chi})$$

$\boldsymbol{\chi}$: vector containing all internal variables of interest, e.g. damage ($\boldsymbol{\Omega}$), hardening variables such as the equivalent plastic strain (γ^p)

Mechanical dissipation inequality (in the absence of plastic porosity changes) :

$$\boldsymbol{\sigma} : \frac{d\epsilon^p}{dt} + \left(\boldsymbol{\sigma} - \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^e}\right)\frac{d\epsilon^e}{dt} + \left(p_l - \frac{\partial \Psi_s}{\partial \phi_l^e}\right)\frac{d\phi_l^e}{dt} + \left(p_g - \frac{\partial \Psi_s}{\partial \phi_g^e}\right)\frac{d\phi_g^e}{dt} - \left(S_s + \frac{\partial \Psi_s}{\partial T}\right)\frac{dT}{dt} - \frac{\partial \Psi_s}{\partial \boldsymbol{\chi}}\frac{d\boldsymbol{\chi}}{dt} \geqslant 0$$

In the absence of irreversible microstructure change...

$$\left(\boldsymbol{\sigma} - \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^e}\right)\frac{d\epsilon^e}{dt} + \left(p_l - \frac{\partial \Psi_s}{\partial \phi_l^e}\right)\frac{d\phi_l^e}{dt} + \left(p_g - \frac{\partial \Psi_s}{\partial \phi_g^e}\right)\frac{d\phi_g^e}{dt} - \left(S_s + \frac{\partial \Psi_s}{\partial T}\right)\frac{dT}{dt} = 0$$

... from which thermodynamic conjugation relationships are deduced...

$$\boldsymbol{\sigma} = \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^e}; \quad p_l = \frac{\partial \Psi_s}{\partial \phi_l^e}; \quad p_g = \frac{\partial \Psi_s}{\partial \phi_g^e}; \quad S_s = -\frac{\partial \Psi_s}{\partial T}$$

... and the reduced dissipation inequality is obtained :

$$\boldsymbol{\sigma} : \frac{d\epsilon^p}{dt} + \boldsymbol{\xi} \cdot \frac{d\boldsymbol{\chi}}{dt} \geqslant 0 \quad ; \quad \boldsymbol{\xi} = -\frac{\partial \Psi_s}{\partial \boldsymbol{\chi}}$$

Tangent Properties [Coussy, 2004]

Expression of the skeleton free energy $\Rightarrow$ REV's tangent properties.

Example : thermo-poro-elasticity (saturated porous medium) with a free energy sought in the form $\Psi_s = \Psi_s(\epsilon^e, p, T)$:

$$\dot{\boldsymbol{\sigma}} = \frac{\partial^2 \Psi_s}{\partial \boldsymbol{\epsilon}^{e2}} : \dot{\boldsymbol{\epsilon}}^e + \frac{\partial^2 \Psi_s}{\partial \boldsymbol{\epsilon}^e \partial p} \dot{p} + \frac{\partial^2 \Psi_s}{\partial \boldsymbol{\epsilon}^e \partial T} \dot{T}$$

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{e}} - \boldsymbol{B}\dot{p} - \alpha \left(\boldsymbol{D} : \boldsymbol{\delta} \right) \dot{T}$$

$\boldsymbol{D}$: stiffness tensor ; $\boldsymbol{B}$: Biot's tensor ; α : thermal expansion coefficient

$$\dot{\phi} = \boldsymbol{b} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{e}} + \frac{\dot{p}}{N} - 3\alpha_\phi \dot{T}$$

$1/N$: inverse of Biot's modulus ; α_ϕ : coefficient for volumetric thermal dilation due to porosity changes only

$$\dot{S}_s = \left(\boldsymbol{D} : \boldsymbol{\delta} \right) \alpha : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{e}} - 3\alpha_\phi \dot{p} + C \frac{\dot{T}}{T}$$

C : skeleton volumetric heat capacity

Flow Properties [Coussy, 2004]

Conduction laws ensure the positivity of the fluid and thermal dissipations in the absence of TH couplings :

- "Darcy's" law :

$$\boldsymbol{v_l} = \phi_l \mathbf{V_l^r} = \frac{\boldsymbol{k_l}}{\eta_l} : [-\boldsymbol{\nabla}_X p_l]$$
$$\Rightarrow \Phi_l = [-\boldsymbol{\nabla}_X p_l] \cdot \boldsymbol{v_l} = \frac{\boldsymbol{k_l}}{\eta_l} : [\boldsymbol{\nabla}_X p_l] : [\boldsymbol{\nabla}_X p_l] \geq 0$$

$$\boldsymbol{v_g} = \phi_g \mathbf{V_g^r} = \frac{\boldsymbol{k_g}}{\eta_g} : [-\boldsymbol{\nabla}_X p_g]$$
$$\Rightarrow \Phi_g = [-\boldsymbol{\nabla}_X p_g] \cdot \boldsymbol{v_g} = \frac{\boldsymbol{k_g}}{\eta_g} : [\boldsymbol{\nabla}_X p_g] : [\boldsymbol{\nabla}_X p_g] \geq 0$$

- Fourier's law :

$$\mathbf{Q} = -\lambda \nabla_X T$$
$$\Rightarrow \Phi_T = \frac{\lambda}{T} \nabla_X T \cdot \nabla_X T \geq 0$$

with positive permeabilities and thermal conductivity

General case of an isothermal solid saturated with water and air

Typical Constitutive Relationships
(hyp. incompressible solid grains, $\boldsymbol{\sigma}_{net} = \boldsymbol{\sigma} - p_g \boldsymbol{\delta}$, $s = p_g - p_l$)

- Stress state law : $\dot{\boldsymbol{\sigma}}_{net} = \boldsymbol{D} : \dot{\boldsymbol{\epsilon}} - \alpha \left(\boldsymbol{D} : \boldsymbol{\delta}\right) \dot{T} - \boldsymbol{\beta} \dot{s}$
- Heat flow equation :
  $\mathbf{Q}_T = -\lambda_T \nabla T + h_{fg} \left(\rho_w \mathbf{V}^*_{vap} + \rho_{vap} \mathbf{V}_a\right)$
  $+ \left[\rho_w C_{Pw} \mathbf{V}_w + \rho_w C_{Pvap} \mathbf{V}^*_{vap} + \rho_a C_{Pa} \mathbf{V}_a\right] (T - T_0)$
- Moisture transfer equation :
  $\mathbf{Q}_{wtot} = \mathbf{Q}_w + \mathbf{Q}_{vap} = \rho_w \left(-\mathbf{D_T} \cdot \nabla T + \mathbf{D_P} \cdot \nabla s - \mathbf{K_w} \cdot \nabla z\right)$
- Air flow equation :
  $$\mathbf{V}_a = -\frac{1}{\gamma_a} \frac{\partial p_a(\mathbf{x}, T(\mathbf{x}))}{\partial T(\mathbf{x})} \mathbf{K_a} \cdot \nabla T(\mathbf{x}) - \mathbf{K_a} \cdot \nabla \left(\frac{p_a}{\gamma_a}\right) - \mathbf{K_a} \cdot \nabla z$$

Balance equations

- Solid skeleton balance equation : $\nabla \cdot \sigma + \mathbf{f} = \mathbf{0}$
- Energy conservation : $\frac{\partial \mathbb{E}_T}{\partial t} + \nabla \cdot \mathbf{Q}_T = 0, \quad \mathbb{E}_T = C_T (T - T_0) + n(1 - S_w) \rho_{vap} h_{fg}$
- Moisture mass conservation : $\frac{\partial \rho_m}{\partial t} + \nabla \cdot \left[\rho_w \left(\mathbf{V}_w + \mathbf{V}^*_{vap}\right)\right] = 0$
- Air mass conservation :
  $\frac{\partial}{\partial t} \left[n \rho_a (1 - S_w + H S_w)\right] + \nabla \cdot \left[\rho_a \mathbf{V}_a\right] + \nabla \cdot \left[\rho_a H \mathbf{V}_w\right] - \nabla \cdot \left[\rho_w \mathbf{V}^*_{vap}\right] = 0$

Choice of the stress state variables for unsaturated porous media

- **number of stress variables ?**
  A long-standing debate [Fredlund and Morgenstein 1977, Houlsby 1997]
  Example for incompressible solid grains :
  - Bishop's effective stress : $\boldsymbol{\sigma}' = (\boldsymbol{\sigma} - p_g \boldsymbol{\delta}) + \chi (p_g - p_l) \boldsymbol{\delta}$
  - independent state variables :
    $(\boldsymbol{\sigma} - p_g \boldsymbol{\delta};\ p_g - p_l)$ or $(p_g - p_l;\ \boldsymbol{\sigma} - p_l \boldsymbol{\delta})$ or $(\boldsymbol{\sigma} - p_l \boldsymbol{\delta};\ \boldsymbol{\sigma} - p_g \boldsymbol{\delta})$

- **nature of the state variables ?**
  No unique formulation [Coussy, 2004 ; Dangla, 2010].
  Example for elastic isothermal unsaturated porous media,
  based on the separation of energies :

$$\Psi_s \left(\boldsymbol{\epsilon}^e, \phi, S_l\right) = \psi_s \left(\boldsymbol{\epsilon}^e, \phi\right) + \phi\, U\left(\phi, S_l\right)$$

$$\begin{cases} \boldsymbol{\sigma} = \dfrac{\partial \psi_s\left(\boldsymbol{\epsilon}^e, \phi\right)}{\partial \boldsymbol{\epsilon}^e} \\[2ex] \pi = S_l p_l + S_g p_g - \dfrac{\partial(\phi\, U(\phi, S_l))}{\partial \phi} = \dfrac{\partial \psi_s\left(\boldsymbol{\epsilon}^e, \phi\right)}{\partial \phi} \\[2ex] p_c = -\dfrac{\partial U\left(\boldsymbol{\epsilon}^e, \phi_l, \phi_g\right)}{\partial S_l} \end{cases}$$

Forms of Geothermal Energy [Lund, 2007]

- comes "from the decay of the naturally occurring isotopes of uranium, thorium and potassium in the earth"
- natural thermal reservoirs above 10km depth : $1.3 \times 10^{27} J \simeq 3 \times 10^{17}$ oil barrels
- geothermal resources $> 150^o C$: electrical power plants
- geothermal resources $< 150^o C$: direct use for heating and cooling

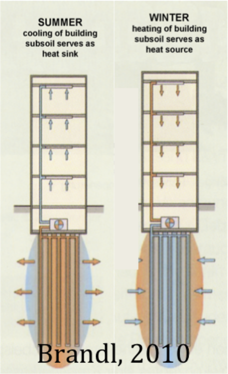
Geothermal Resources

Resource Type	Temperature Range (°C)
Convective hydrothermal resources	
Vapor dominated	240°
Hot-water dominated	20 to 350°+
Other hydrothermal resources	
Sedimentary basin	20 to 150°
Geopressured	90 to 200°
Radiogenic	30 to 150°
Hot rock resources	
Solidified (hot dry rock)	90 to 650°
Part still molten (magma)	>600°

Direct Use of Geothermal Energy (2005)

Country	GWh/yr	MWt	Main Applications
China	12,605	3,687	bathing
Sweden	12,000	4,200	GHP
USA	8,678	7,817	GHP
Turkey	6,900	1,495	district heating
Iceland	6,806	1,844	district heating
Japan	2,862	822	bathing (onsens)
Hungary	2,206	694	spas/greenhouses
Italy	2,098	607	spas/space heating
New Zealand	1,969	308	industrial uses

GCHP with Heat Exchanger Piles



- ground = heat reservoir
- high thermal conductivity of ground and grout

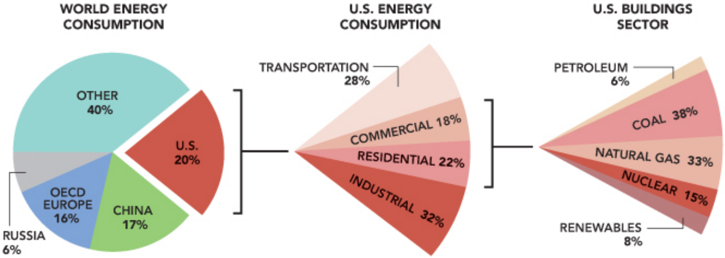
  ⇒ foundation piles used as heat exchangers

  ⇒ thermo-mechanical behavior of the foundation ?
- heat stored in summer, pumped in winter

  ⇒ annual energy balance ?

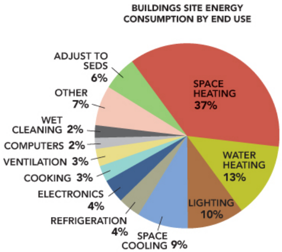
  ⇒ design and performance ?

Why using GCHP in buildings ? [DoE Buildings Data Book]

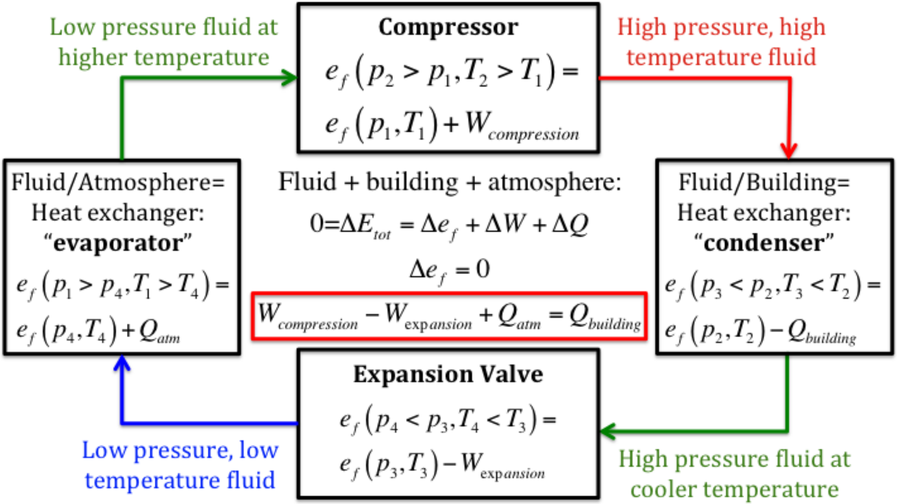


- 73% of U.S. electrical energy
- 55% of U.S. natural gas

consumed by U.S. buildings in 2008, mainly for heating and lighting



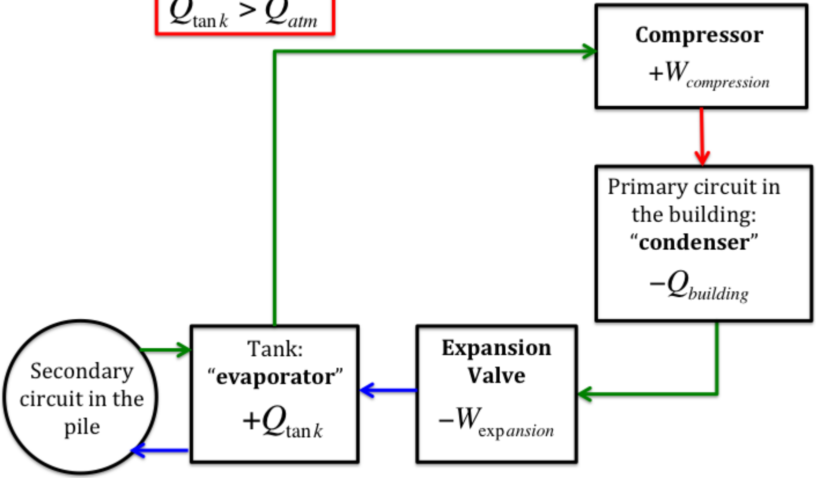
Thermodynamic Principle of a Heat Pump



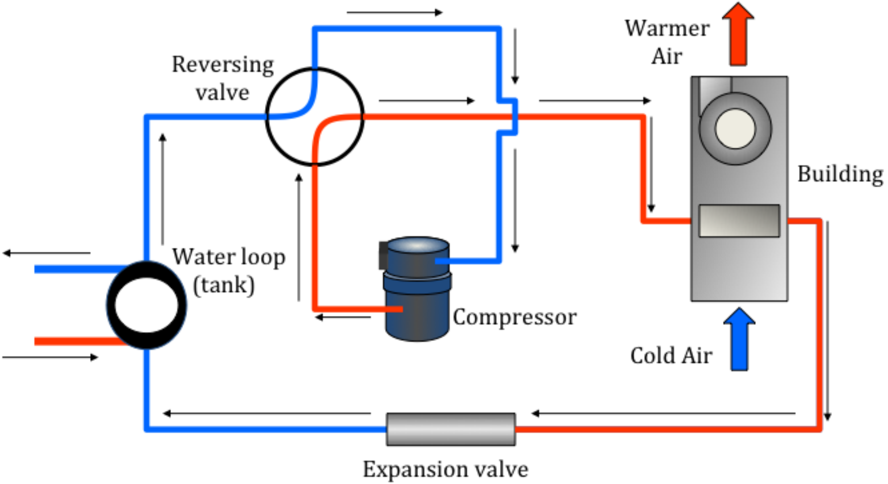
Thermodynamic Balance of a GCHP

$$W_{compression} - W_{expansion} + Q_{tank} = Q_{building}$$

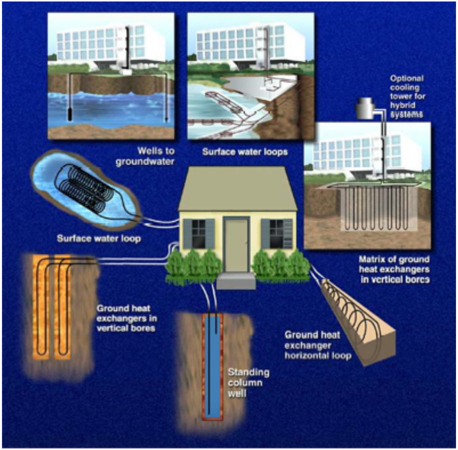
$$\boxed{Q_{tank} > Q_{atm}}$$



Typical Earth-Coupled HVAC System [Akrouch, 2011]



Examples of GCHP Systems [Hughes, 2008 (Oak Ridge Nat. Lab.)]

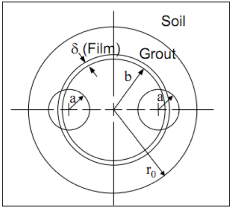


In average, a U.S. household consumes 8,900 kWh of electrical energy per year.

Cooling down by 1^oC the volume of soil below a typical one-family house (100 m^2) over a pile depth of 12m could bring up to 1,000 kWh.

Preliminary Numerical Results [Berns and Arson, 2011]

Equivalent axis-symmetric model inspired from work done at Oakridge National Laboratory [Shonder and Beck, 2000].



Simulation with Theta-Stock FEM code [Gatmiri and Arson, 2008].

- 1D mesh, i.e. fixed depth
- 3 thermo-elastic materials : grout, film, soil

$$\boldsymbol{\sigma} = \frac{E}{(1+\nu)}\epsilon + \frac{E}{(1+\nu)}\frac{\nu}{(1-2\nu)} Tr(\epsilon)\,\boldsymbol{\delta}$$

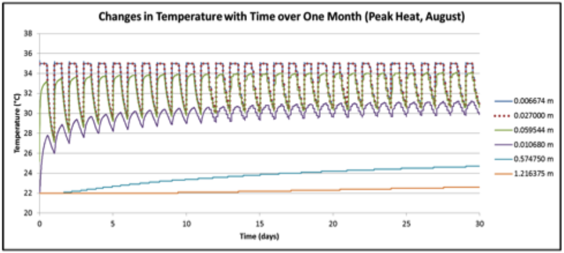
$$+\frac{E}{3(1-2\nu)}(1-n)\alpha_s \Delta T \boldsymbol{\delta}$$

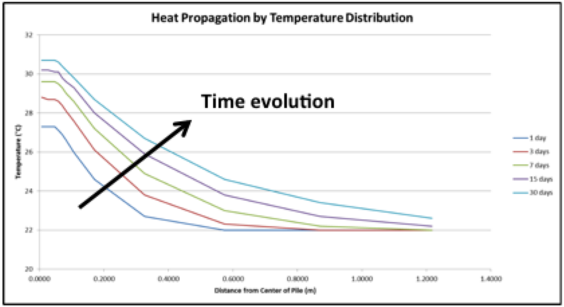
$$\boldsymbol{q} = -(1-n)\lambda_s \boldsymbol{\nabla} T$$

- film : fictitious material with mechanical properties of the grout and thermal properties of water
- free heating and cooling : temperature imposed in the film $\simeq$ typical atmospheric temperature recorded in Texas

Thermal Influence Zone in Cooling Mode

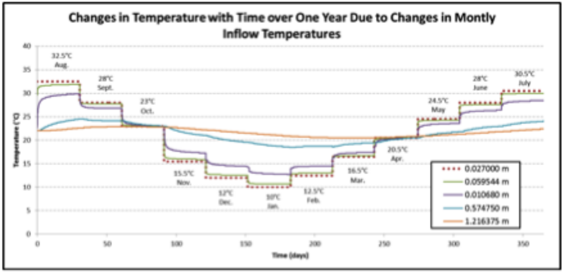
- initial temperature of the ground : $22\,^oC$
- pile diameter : 1.1m
- thermal zone of influence > 1 pile diameter

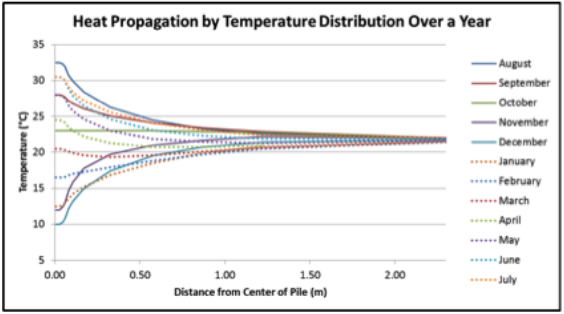




1-year cycle for typical Texan temperatures

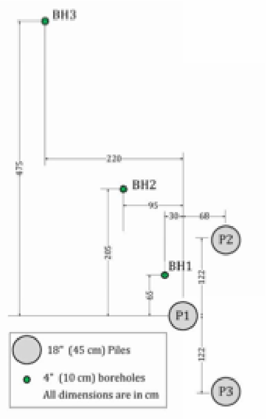
- initial temperature of the ground : $22\,^oC$
- pile diameter : 1.1m
- thermal zone of influence > 1 pile diameter
- ground heating, no energy balance



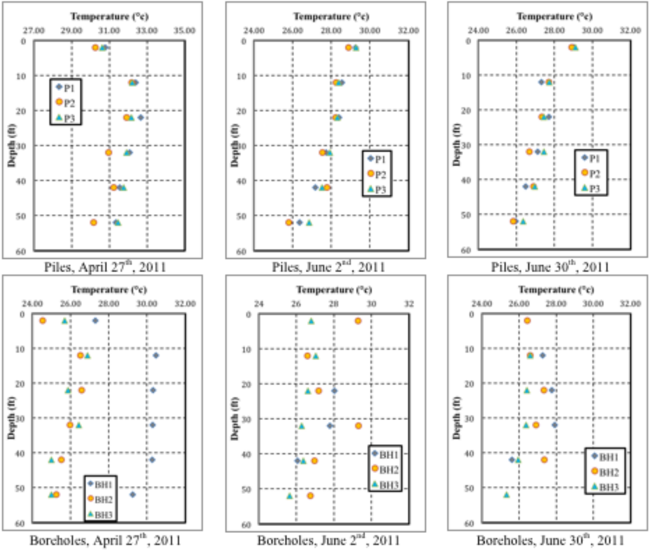


TAMU Liberal Arts Building Site [Briaud, Akrouch, Arson, Sanchez]

- foundation piles of the Liberal Arts Building : 20 feet deep below the pile cap
- reinforcement : steel cage of 6 #6 rebars + a single # 9 steel rebar in the middle
- 3 piles equipped with a High Density Polyethylene (HDPE) U-shaped tube
- average distance between the HDPE pipes along the pile : 20 cm
- 6 thermistors in each experimental pile + in 3 close boreholes (at same depths)

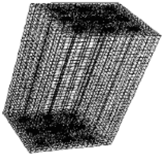


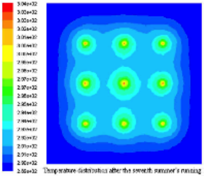
Preliminary Experimental Results



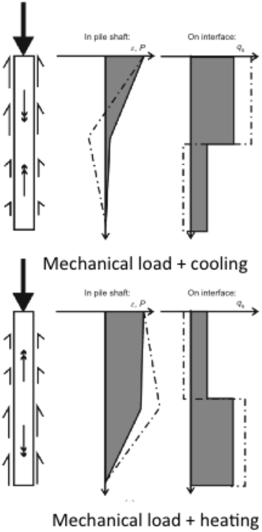
Research Prospectives

- techniques to measure ground thermal properties in the field
- full scale heating and cooling tests on the foundation of the Liberal Arts Building
- influence of thermal cyclic loading on the thermo-hydro-mechanical behavior of unsaturated expansive clays and on soil/structure interactions
- energy pile group effects in cooling mode
- cost assessment





[Li et al., 2010]



[Bourne-Webb et al., 2009]

Framework of Fracture Networks

Multimodal models [Pruess et al. 1990]

- several connected networks forming a unique equivalent network (e.g. pores of the intact matrix + cracks)
- 1 pressure head h for the whole network, 1 single balance equation

$$\frac{\partial \theta_w(h)}{\partial h}\frac{\partial h}{\partial t} = \frac{\partial}{\partial \boldsymbol{x}} \cdot \boldsymbol{K}_{\boldsymbol{w}}(h) \cdot \frac{\partial}{\partial \boldsymbol{x}}(h+z)$$

- homogenized, equivalent retention properties $\theta_w(h)$ and $k_R(h)$

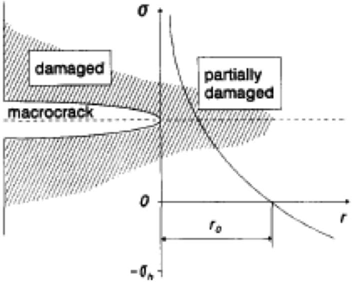
$$\begin{cases} \theta_w(h) = w_f\,\theta_{w,f}(h) + (1-w_f)\,\theta_{w,m}(h) \\ k_R(h) = w_f\,k_{R,f}(h) + (1-w_f)\,k_{R,m}(h) \end{cases}$$

Multicontinuum systems

- several networks behaving as separate entities
- coupled balance equations [Gwo et al. 1995, Vogel et al. 2000]
- permeability : retention properties ? equivalent pressure head ? [Gerke and Van Genuchten 1993, Zimmermann et al. 1996]

Framework of Fracture Mechanics

- study of the initiation, propagation and coalescence of cracks
- initiation and propagation criteria based on energy balance at crack tips
- requires the determination of stress intensity factors (e.g., Griffith theory)



[Valko and Economides, 1994]

Limitations : scale dependence, no direct relation to elastic properties at the REV scale

An alternative approach based on a combination of fracture and damage mechanics :
characterization of a damaged zone around cracks with a damaged (effective) stress variable
[Valko and Economides, 1994]

Framework of Continuum Damage Mechanics

Micro-mechanical models

- effective material area decrease, redistribution of stresses [Kachanov, 1992]
- stress in the intact solid matrix $\neq$ stress in the cracked material, effective stress state variables apply to a fictitious undamaged system counterpart [de Borst et al. 1999]
- the size of the Representative Elementary Volume needs to be defined to homogenize the fields of variables and the damaged properties [Nemat-Nasser and Hori, 1994 ; Dormieux et al., 2006]

Phenomenological models

1. "averaged" variables, directly defined at the REV scale : damage = crack density tensor (often) ; effective stress (applying to the undamaged matrix)
2. reduced dissipation inequality obtained from the *Inequality of Clausius-Duhem*

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{e}} - S_s\, dT - \dot{\Psi}_s\left(\boldsymbol{\epsilon}^{\boldsymbol{e}}, T, \boldsymbol{\Omega}\right) \geq 0 \quad \Longrightarrow \boldsymbol{Y} : \dot{\boldsymbol{\Omega}} \geq 0$$

3. stress/strain relations derived from the expression of the free energy

Formulation of Phenomenological Damage Models

1. "averaged" variables, directly defined at the REV scale

   Crack density tensor, sufficient to characterize the influence of damage on stiffness is cracks do not interact [Kachanov, 1992] :

$$\boldsymbol{\Omega} = \boldsymbol{D_c} = \frac{1}{V_{REV}} \sum_{k=1}^{N} \left(l^k\right)^3 \boldsymbol{n^k} \otimes \boldsymbol{n^k}$$

   Effective stress defined with a damage operator [Hansen and Schreyer 1994]

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{M}(\boldsymbol{\Omega}) : \boldsymbol{\sigma}$$

2. reduced dissipation inequality obtained from the *Inequality of Clausius-Duhem*

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{e}} - S_s\, dT - \dot{\Psi}_s\left(\boldsymbol{\epsilon}^{\boldsymbol{e}}, T, \boldsymbol{\Omega}\right) \geq 0 \quad \Longrightarrow \boldsymbol{Y} : \dot{\boldsymbol{\Omega}} \geq 0$$

3. stress/strain relations derived from the expression of the free energy, in particular :

$$\boldsymbol{\sigma} = \frac{\partial \Psi_s\left(\boldsymbol{\epsilon}^{\boldsymbol{e}}, T, \boldsymbol{\Omega}\right)}{\partial \boldsymbol{\epsilon}^{\boldsymbol{e}}} = \boldsymbol{D}(\boldsymbol{\Omega}) : \boldsymbol{\epsilon}^{\boldsymbol{e}} - \alpha\left(\boldsymbol{D}(\boldsymbol{\Omega}) : \boldsymbol{\delta}\right) T$$

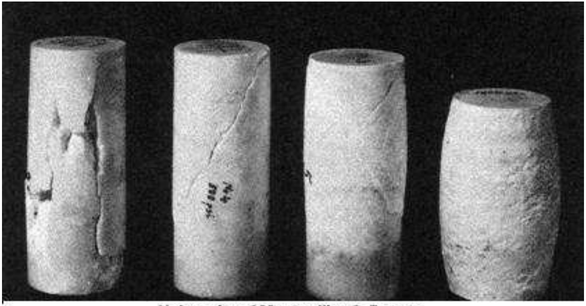
4. the Principle of Equivalent Elastic Energy is often used to determine $\boldsymbol{D}(\boldsymbol{\Omega})$:

$$\Psi_s\left(\boldsymbol{\epsilon}^{\boldsymbol{e}}, T, \boldsymbol{\Omega}\right) + \Psi_s^*\left(\boldsymbol{\sigma}, T, \boldsymbol{\Omega}\right) = \boldsymbol{\sigma} : \boldsymbol{\epsilon}^{\boldsymbol{e}}$$

$$\Psi_s^*\left(\boldsymbol{\sigma}, T, \boldsymbol{\Omega}\right) = \Psi_s^*\left(\tilde{\boldsymbol{\sigma}}, T, \boldsymbol{\Omega} = \boldsymbol{0}\right) \Rightarrow \boldsymbol{D}(\boldsymbol{\Omega}) = \boldsymbol{M}(\boldsymbol{\Omega})^{-1} : \boldsymbol{D_0} : \boldsymbol{M}(\boldsymbol{\Omega})^T$$

A Lack of Experimental Data

- lab protocols available for mechanical damage parameters [Halm and Dragon, 2002] and poro-elastic parameters [Gatmiri, 1997]
- determination of retention properties from the Pore Size Distribution curve [Romero and Jommi, 2008] : bimodal porosity, but for undamaged material
- permeability assessment based on scalar variables (no flow orientation) [Dal Pont et al., 2004], or on postulates on the cracks shape and density [Shao et al., 2005 ; Maleki and Pouya, 2010]



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Limitations of Models of Damaged Tangent Properties

Damage in "unsaturated" behaviour laws

- Bishop's effective stress concept [Shao et al. 2005]

$$\sigma'_{ij} = \sigma^{dry}_{ij} - b\,[S_w\, p_w + (1 - S_w)\, p_a]\;\delta_{ij}$$

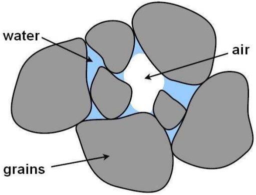
$$\sigma'_{ij} = \frac{\partial \Psi_s(\epsilon,\Omega)}{\partial \epsilon} - b\,[S_w\, p_w + (1 - S_w)\, p_a]\;\delta_{ij}$$

no consensus on the dependence of b to damage

- only 1 damage model formulated in independent state variables [Lu et al. 2006]

no assessment of the model thermodynamic consistency

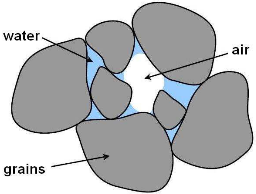
- recent developments on chemo-thermal processes in cementitious materials based on a scalar damage variable [Gawin et al., 2003 ; Schrefler and Pesavento, 2004]



Limitations of Models of Damaged Transport Properties

Damage in fluid transfer laws

- *Fracture network theories.*
  Relative permeability computed by integration of retention variables. Intrinsic permeability assessed for a fixed porous network (rigid solid skeleton, no damage evolution)
  $\Rightarrow$ *uncoupled, purely "hydraulic" approach*
- *Coupled poro-elastic models based on state surfaces.* [Gatmiri, 1997-2008]
  $\Rightarrow$ *no damage*
- *Existing continuum damage models.*
  Biot's effective stress [Yang et al. 2007]. Micro-flow homogenization [Shao et al. 2005].



Damage is non local

Damage is non local, i.e. $\Omega(\boldsymbol{x})$ influences fields of variables at $\boldsymbol{x} + \boldsymbol{dx}$ [Bazant, 1991]

- integral formulations : usually, non-local deformation or non-local energy release rate [Jirasek, 1998]

$$\bar{f}(x) = \int_{V_{tot}} \alpha(x,\xi) f(\xi) d\xi, \qquad \alpha(x,\xi) = \frac{\alpha_\infty(\|x-\xi\|)}{\int_{V_{tot}} \alpha_\infty(\|x-\xi\|) d\xi}$$

- differential formulations : usually, introduction of the gradient of deformation [Askes and Sluys, 2002] or the gradient of damage [Frémond and Nedjar, 1996]

$$\epsilon(x+s) = \epsilon(x) + s\,\frac{d\epsilon(x)}{dx} + s^2\,\frac{d^2\epsilon(x)}{dx^2} + o(s^2), \qquad \bar{\epsilon}(x) = \frac{1}{l}\int_{-l/2}^{+l/2} \epsilon(x+s) ds$$

$$\Rightarrow \bar{\epsilon}(x) = \epsilon(x) + \frac{l^2}{24}\,\frac{d^2\epsilon(x)}{dx^2} + o(l^2)$$

- microstructure-enriched models [Mindlin, 1964 ; Germain, 1973(a,b)] : usually second-gradient models (micro-translation), Cosserat models (micro-rotation)

Characteristic length in the expression of the free energy

residual crack openings scaled by crack surface energy
fracture energy stored / square of the characteristic length

- energetic term for residual strains after unloading [Halm and Dragon, 1998]

$$\Psi\left(\epsilon,\Omega\right) = \frac{1}{2}\epsilon : \boldsymbol{D}\left(\Omega\right) : \epsilon - g\boldsymbol{\Omega} : \boldsymbol{\epsilon}$$

- surface energy due to crack opening [Hansen and Schreyer, 1994]

$$\Psi\left(\epsilon,\Omega,\Omega^{\boldsymbol{h}}\right) = \frac{1}{2}\epsilon : \boldsymbol{D}\left(\Omega\right)\,\epsilon + H\left(\Omega^{\boldsymbol{h}}\right) + \gamma_D\,\boldsymbol{\Omega} : \boldsymbol{\Omega}$$

- energy related to the zone of influence of damage
  [Frémond and Nedjar, 1998]

$$\Psi\left(\epsilon,\omega,\nabla\omega\right) = \frac{1}{2}\epsilon : \boldsymbol{D_0} : \epsilon + W\left(1-\omega\right) - M\left[log(|\omega|) - \omega + 1\right] + \frac{k}{2}\left(\nabla\omega\right)^2$$

Modeling Viscoplastic Damage in Saturated Rock [Dufour et al., 2011]

Objective : modeling creep processes in the Excavation Damaged Zone

Framework : isothermal saturated rock

- Helmholtz free energy for the solid skeleton : $\Psi_s\left(\boldsymbol{\epsilon^e}, \Phi^e, \boldsymbol{\Omega}, \gamma^{vp}\right)$:

$$\boldsymbol{\sigma} : \left(\dot{\boldsymbol{\epsilon}}^{\boldsymbol{e}} + \dot{\boldsymbol{\epsilon}}^{\boldsymbol{vp}}\right) + p_w \left(\dot{\Phi}^e + \dot{\Phi}^{vp}\right) - \dot{\Psi}_s \geq 0$$

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{vp}} + p_w \dot{\Phi}^{vp} + \boldsymbol{Y} : \dot{\boldsymbol{\Omega}} + k\,\dot{\gamma}^{vp} \geq 0$$

$$\boldsymbol{\sigma} = \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon^e}}, \quad p_w = \frac{\partial \Psi_s}{\partial \Phi^e}, \quad \boldsymbol{Y} = -\frac{\partial \Psi_s}{\partial \boldsymbol{\Omega}}, \quad k = -\frac{\partial \Psi_s}{\partial \gamma^{vp}}$$

- convex, positive-definite dissipation potential sought in the form $\phi\left(\boldsymbol{\sigma}, p_w, \boldsymbol{Y}, k\right)$, with :

$$\dot{\boldsymbol{\epsilon}}^{vp} = \frac{\partial \phi}{\partial \boldsymbol{\sigma}}, \quad \dot{\Phi}^{vp} = \frac{\partial \phi}{\partial p_w}, \quad \dot{\boldsymbol{\Omega}} = -\frac{\partial \phi}{\partial \boldsymbol{Y}}, \quad \dot{\gamma}^{vp} = -\frac{\partial \phi}{\partial k}$$

Problem : what is the driving force in the dissipation potential ? (1/3)

- in Continuum Damage Mechanics :

  Principle of Equivalent Elastic Energy :

$$\Psi_s\left(\epsilon^{\boldsymbol{e}}, \Omega | \boldsymbol{\sigma}\right) = \frac{1}{2}\epsilon^{\boldsymbol{e}} : \boldsymbol{D}\left(\Omega\right) : \epsilon^{\boldsymbol{e}} = \frac{1}{2}\boldsymbol{\sigma}^T : \boldsymbol{D}^{-T}\left(\Omega\right) : \boldsymbol{\sigma}$$

$$= \frac{1}{2}\tilde{\boldsymbol{\sigma}}^T : \boldsymbol{D}_{\boldsymbol{0}}^{-T} : \tilde{\boldsymbol{\sigma}} = \Psi_s\left(\epsilon^{\boldsymbol{e}}, \Omega = \boldsymbol{0} | \tilde{\boldsymbol{\sigma}}\right)$$

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{M}\left(\Omega\right) : \boldsymbol{\sigma}$$

- in Damage Mechanics coupled to poro-elasticity [Coussy, 2004] :

$$\boldsymbol{\sigma} = \boldsymbol{D}\left(\Omega\right) : \epsilon^{\boldsymbol{e}} - p_w\, \boldsymbol{B}\left(\Omega\right)$$

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p_w\, \boldsymbol{B}\left(\Omega\right) = \boldsymbol{D}\left(\Omega\right) : \epsilon^{\boldsymbol{e}}$$

Problem : what is the driving force in the dissipation potential ? (2/3)

...in Damage Mechanics coupled to poro-elasticity...

- ... **either** a decomposition of the free energy is postulated...

$$\Psi_s^* \left(\epsilon^{\boldsymbol{e}}, p_w, \boldsymbol{\Omega} \right) = \Psi_{s1}^* \left(\epsilon^{\boldsymbol{e}}, \boldsymbol{\Omega} \right) + \Psi_{s2}^* \left(p_w \right)$$

... and Biot's tensor can be expressed as a function of the drained damaged elastic tensor...

$$\boldsymbol{B}\left(\boldsymbol{\Omega}\right) = \boldsymbol{\delta} - \frac{1}{3\,K_s} \boldsymbol{D}\left(\boldsymbol{\Omega}\right) : \boldsymbol{\delta}$$

- ... **or** this decomposition is not postulated...
  ... and Biot's tensor is expressed as a function of the undrained damaged stiffness tensor [Shao, 1998]

Problem : what is the driving force in the dissipation potential ? (3/3)

- in poro-(visco)-plasticity, it is often postulated that :

$$\dot{\Phi}^{vp} = \boldsymbol{B_0} : \dot{\epsilon}^{vp}$$

As a result :

$$\boldsymbol{\sigma'} = \boldsymbol{\sigma} + p_w \boldsymbol{B_0}$$

- in Damage Poro-(visco)-plasticity, should it be...

$$\boldsymbol{\sigma_{eff}} = \boldsymbol{M}(\Omega) : \boldsymbol{\sigma} + p_w \boldsymbol{B_0} \quad ?$$

$$\boldsymbol{\sigma_{eff}} = \boldsymbol{M}(\Omega) : (\boldsymbol{\sigma} + p_w \boldsymbol{B_0}) \quad ?$$

something else ? ? ?

Modeling Approach : Concept of Double Effective Stress

- Dependence of the visco-plastic porosity change rate to damage :

$$\dot{\Phi}^{vp} = \boldsymbol{B}(\Omega) : \dot{\epsilon}^{vp}$$

- (Classical) decomposition of the free energy of the solid skeleton :

$$\Psi_s^*\left(\epsilon^{\boldsymbol{e}}, p_w, \Omega, \gamma^{vp}\right) = \Psi_{s1}^*\left(\epsilon^{\boldsymbol{e}}, p_w, \Omega\right) + \Psi_{s2}^*\left(\gamma^{vp}\right)$$

Modeling Approach : Concept of Double Effective Stress

- Dependence of the visco-plastic porosity change rate to damage :

$$\dot{\Phi}^{vp} = \boldsymbol{B}(\boldsymbol{\Omega}) : \dot{\boldsymbol{\epsilon}}^{vp}$$

- (Classical) decomposition of the free energy of the solid skeleton :

$$\Psi_s^*\left(\boldsymbol{\epsilon^e}, p_w, \boldsymbol{\Omega}, \gamma^{vp}\right) = \Psi_{s1}^*\left(\boldsymbol{\epsilon^e}, p_w, \boldsymbol{\Omega}\right) + \Psi_{s2}^*\left(\gamma^{vp}\right)$$

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{vp}} + p_w\,\dot{\Phi}^{vp} + \boldsymbol{Y} : \dot{\boldsymbol{\Omega}} + k\,\dot{\gamma}^{vp} \geq 0$$

$$\left(\boldsymbol{\sigma} + p_w\boldsymbol{B}(\boldsymbol{\Omega})\right) : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{vp}} + \boldsymbol{Y} : \dot{\boldsymbol{\Omega}} + k\,\dot{\gamma}^{vp} \geq 0$$

$$\boldsymbol{\sigma'} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{vp}} + \boldsymbol{Y} : \dot{\boldsymbol{\Omega}} + k\,\dot{\gamma}^{vp} \geq 0$$

"double effective stress"

$$\Psi_s^*\left(\boldsymbol{\epsilon^e}, p_w, \boldsymbol{\Omega}, \gamma^{vp}\right) = \Psi_{s1}^*\left(\boldsymbol{\epsilon^e}, \boldsymbol{\Omega}\,|\boldsymbol{\sigma'}\right) + \Psi_{s2}^*\left(\gamma^{vp}\right)$$

$$\Psi_s^*\left(\boldsymbol{\epsilon^e}, p_w, \boldsymbol{\Omega}, \gamma^{vp}\right) = \Psi_{s1}^*\left(\boldsymbol{\epsilon^e}, |\boldsymbol{M}(\boldsymbol{\Omega}) : \boldsymbol{\sigma'}\right) + \Psi_{s2}^*\left(\gamma^{vp}\right)$$

$$\boldsymbol{\sigma_{eff}} = \boldsymbol{M}(\boldsymbol{\Omega}) : \left(\boldsymbol{\sigma} + p_w\boldsymbol{B}(\boldsymbol{\Omega})\right)$$

$$\boldsymbol{\sigma_{eff}} : \dot{\boldsymbol{\epsilon}}^{\boldsymbol{vp}} + k\,\dot{\gamma}^{vp} \geq 0, \quad \dot{\boldsymbol{\epsilon}}^{vp} = \frac{\partial\phi}{\partial\boldsymbol{\sigma_{eff}}}$$

$\boldsymbol{\sigma_{eff}}$ is "conjugate" to the viscoplastic strain rate by the dissipation potential.

Independent State Variables

1. **Assumption :** incompressible solid phase. Clausius-Duhem Inequality :

$$(\sigma_{ij} - p_a\delta_{ij})\ \Delta\epsilon_{ji} + (p_a - p_w)\Delta(-nS_w) - S\Delta T - \Delta\Psi_s(\epsilon_{ij}, nS_w, T, \Omega_{ij}) \geq 0$$

2. **3 independent strain variables :** mechanical strain $\epsilon_{M_{ij}}$, capillary strain ϵ_{Sv} and thermal strain ϵ_{Tv}...
...conjugate to 3 independent stress variables :
net stress $\sigma"_{ij} = \sigma_{ij} - p_a\delta_{ij}$, suction $s = p_w - p_a$, and thermal stress p_T :

$$\begin{cases} \sigma"_{ij} \leftrightarrow \epsilon_{M\,ij} \\ s \leftrightarrow \epsilon_{Sv} \\ p_T \leftrightarrow \epsilon_{Tv} \end{cases}$$

3. **Thermodynamic decomposition of the total strain tensor :**

$$d\epsilon_{ij} = d\epsilon^e_{M_{ij}} + d\epsilon^d_{M_{ij}} + \frac{1}{3}\delta_{ij}\left(d\epsilon^e_{Sv} + d\epsilon^d_{Sv}\right) + \frac{1}{3}\delta_{ij}\left(d\epsilon^e_{Tv} + d\epsilon^d_{Tv}\right)$$

e : elastic, d : non-elastic

4. Clausius-Duhem Inequality written in terms of **stress/strain products :**

$$\sigma"_{ij}\,\Delta\epsilon_{M_{ij}} + s\,\Delta\epsilon_{Sv} + p_T\,\Delta\epsilon_{Tv} - \Delta\Psi_s\left(\epsilon_{M_{ij}}, \epsilon_{Sv}, \epsilon_{Tv}, \Omega_{ij}\right) \geq 0$$

Stress/Strain Relationships

- 3 components of Helmholtz free energy :

$$\Psi_s(\epsilon_{M_{kl}}, \epsilon_{Sv}, \epsilon_{Tv}, \Omega_{kl}) =$$

$$\frac{1}{2}\epsilon_{M_{ji}}\, D_{e_{ijkl}}\,(\Omega_{pq})\,\epsilon_{M_{lk}} + \frac{1}{2}\epsilon_{Sv}\beta_s\,(\Omega_{pq})\,\epsilon_{Sv} + \frac{1}{2}\epsilon_{Tv}\beta_T\,(\Omega_{pq})\,\epsilon_{Tv}$$

$$-g_M\,\Omega_{ij}\,\epsilon_{M_{ji}} - \frac{g_S}{3}\,\delta_{ij}\,\Omega_{ji}\,\epsilon_{Sv} - \frac{g_T}{3}\,\delta_{ij}\,\Omega_{ji}\,\epsilon_{Tv}$$

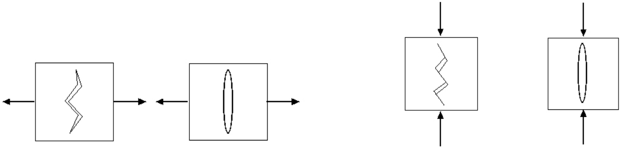
- Conjugation Relationships :

$$\sigma''_{ij} = \frac{\partial\Psi_s(\epsilon_{M_{kl}},\epsilon_{Sv},\epsilon_{Tv},\Omega_{kl})}{\partial\epsilon_{M_{ij}}}, \quad s = \frac{\partial\Psi_s(\epsilon_{M_{kl}},\epsilon_{Sv},\epsilon_{Tv},\Omega_{kl})}{\partial\epsilon_{Sv}}$$

$$p_T = \frac{\partial\Psi_s(\epsilon_{M_{kl}},\epsilon_{Sv},\epsilon_{Tv},\Omega_{kl})}{\partial\epsilon_{Tv}}, \quad Y_{d\,ij} = -\frac{\partial\Psi_s(\epsilon_{M_{kl}},\epsilon_{Sv},\epsilon_{Tv},\Omega_{kl})}{\partial\Omega_{ij}}$$

- Damaged Rigidities Computed by the Principle of Equivalent Elastic Energy [Codebois et Sidoroff 1982]

Damage Evolution Law



- influence of tensile mechanical stress, thermal expansion and capillary pore shrinkage :

$$Y^{+}_{d1\,ij} = g_M\,\epsilon^{+}_{M\,ij} + \frac{g_S}{3}\epsilon^{-}_{Sv}\delta_{ij} + \frac{g_T}{3}\epsilon^{+}_{Tv}\delta_{ij}$$

- a unique damage criterion [Dragon et Halm 1996] :

$$f_d\left(Y_{d\,pq}, \Omega_{pq}\right) = \sqrt{\frac{1}{2}\,Tr\left(Y^{+}_{d_1\,ij}\,Y^{+}_{d_1\,ji}\right)} - C_0 - C_1\delta_{ij}\,\Omega_{ji}$$

- after applying the consistency rules :

$$d\Omega_{ij} = d\lambda_d \frac{\partial f_d\left(Y_{d\,pq}, \Omega_{pq}\right)}{\partial Y^{+}_{d_1\,ji}}$$

Transfer Equations

Liquid Water Flow :

$$V_{w_i} = -\frac{\Psi_R(\theta_w)}{\sigma(T_{ref})}\frac{d\sigma(T)}{dT} K_{w_{ij}} \nabla(T)_j + \frac{1}{\gamma_w}\frac{\sigma(T)}{\sigma(T_{ref})} K_{w_{ij}} \nabla(s)_j - K_{w_{ij}} \nabla(z)_j$$

Influence of Damage on the **intrinsic permeability** :

$$K_{w_{ij}} = k_r(S_w,\ T)\, K_{int_{ij}}(n, \Omega_{pq})$$

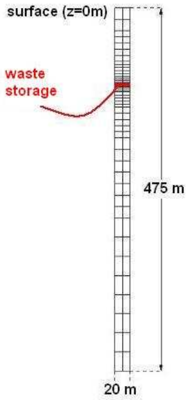
$$K_{w_{ij}} = k_r(S_w,\ T)\left[K_{ij}^{intact}(n^{rev}) + K_{ij}^{dg}(n^{frac}, \Omega_{pq})\right]$$

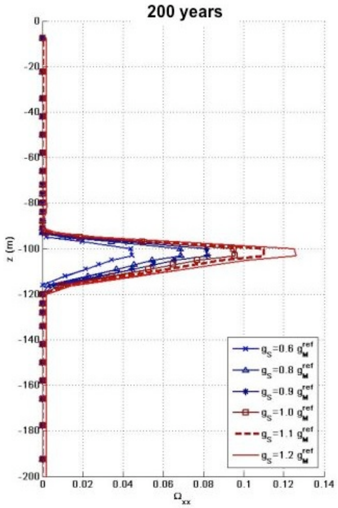
Assumption : laminar flow in the 3 equivalent cracks of the REV [Shao et al. 2005] :

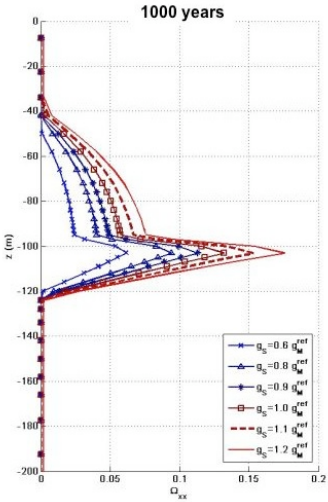
$$K_{ij}^{dg}\left(n^{frac}, \Omega_{rs}\right) = \frac{\pi^{-2/3}\gamma_w}{12\,\mu_w(T_{ref})}\chi^{4/3}\, b^2 \sum_{k=1}^{3}\left(d^k\right)^{5/3}\left(\delta_{ij} - n_i^k\, n_j^k\right)$$

b : internal length parameter

Influence of the Damage Parameters [Pollock 1986]







$S_{w0} = 0.15$

Influence of the Internal Length Parameter [Pollock 1986]

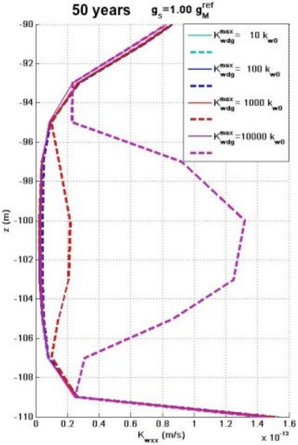
$$K^{dg}_{2_{ij}}\left(n^{frac}, \Omega_{rs}\right) =$$

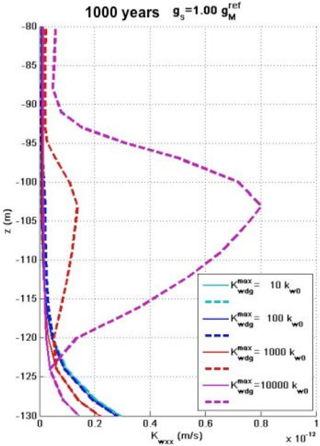
$$\frac{\pi^{-2/3}\,\gamma_w}{12\,\mu_w(T_{ref})}\,\chi^{4/3}\,b^2$$

$$\times \sum_{k=1}^{3}\left(d^k\right)^{5/3}\left(\delta_{ij} - n^k_i\, n^k_j\right)$$

$K^{dg}_{ij} = K^{max}_{w,\,dg}\,\delta_{ij}$ for $\Omega_{ij} = 0.95\delta_{ij}$

$\Rightarrow$ computation of b





Main Assumptions [Arson and Pereira, 2011]

- Cracks do not interact ⇒ damage = crack-density tensor [Kachanov, 1992]

$$\boldsymbol{\Omega} = \sum_{k=1}^{3} d^k \boldsymbol{n^k} \otimes \boldsymbol{n^k}$$

- Cracks do not intersect but are connected to the natural pores :

$$\boldsymbol{K_w} = \boldsymbol{K_w^0} + \boldsymbol{K_w^c}$$

- Cracks and natural pores are connected, but do not overlap :

$$V_v = V_p + V_c$$

- Flow in the natural pores and cracks is modeled as a laminar flow in parallel cylinders. For an isotropic model [Garcia-Bengochea et al., 1979] :

$$k_w = \frac{\gamma}{8\mu} \Phi \frac{1}{\int_0^\infty f(r) dr} \int_0^\infty f(r) r^2 dr$$

Size Distributions of the Natural Pores and Cracks (1/3)

Volumetric frequency of the pores of radius r :

$$f(r) = L\,\alpha(r)\,\pi\, r^2 \qquad \alpha(r) = \alpha_p(r) + \alpha_c(r)$$

Micro/macro relationship for a unit REV ($L = 1$) :

$$\int_{r_{min}^{p}}^{r_{max}^{p}} \alpha_p(r)\pi r^2 dr = V_p \qquad \int_{r_{min}^{c}}^{r_{max}^{c}} \alpha_c(r)\pi r^2 dr = V_c$$

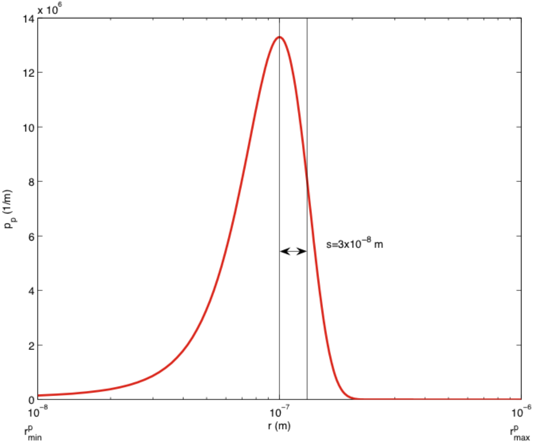
r_{min}^{p}, r_{max}^{p}, r_{min}^{c}, r_{max}^{c} : min. and max. radius values for natural pores and cracks
α_p : frequency of occurrence of natural pores of radius r in the REV
α_c : frequency of occurrence of cracks of radius r in the REV

$$\pi\, N_p \int_{r_{min}^{p}}^{r_{max}^{p}} p_p(r) r^2 dr = V_p \qquad \pi\, N_c \int_{r_{min}^{c}}^{r_{max}^{c}} p_c(r) r^2 dr = V_c$$

N_p, N_c : number of natural pores and cracks in the REV

Size Distributions of the Natural Pores and Cracks (2/3)

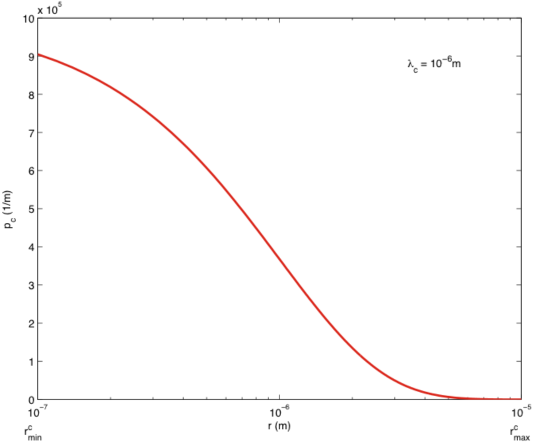
Natural Pores : bell-shaped distribution [Van Genuchten, 1980 ; Alves et al., 1996]



$$p_p(r) = \begin{cases} \frac{1}{s\sqrt{2\pi}} exp\left(-\frac{(r-m)^2}{2\,s^2}\right) & if \quad r^p_{min} \leq r \leq r^p_{max} \\ 0 \quad if \quad r < r^p_{min} \quad or \quad if \quad r > r^p_{max} \end{cases}$$

Size Distributions of the Natural Pores and Cracks (3/3)

Cracks : exponential distribution [Maleki, 2004]



$$p_c(r) = \begin{cases} \frac{1}{\lambda_c} exp\left(-\frac{r}{\lambda_c}\right) & if \quad r^c_{min} \leq r \leq r^c_{max} \\ 0 \quad if \quad r < r^c_{min} \quad or \quad if \quad r > r^c_{max} \end{cases}$$

Updating k_w with the State Variables

$$k_w = \frac{\gamma}{8\mu}\Phi \frac{1}{\int_0^\infty f(r)dr} \int_0^\infty f(r) r^2 dr$$

Knowing the State Variables ϵ_{ij} and Ω_{ij} at the current iteration :

1. updating porosity Φ (hyp. incompressible solid grains) :

$$-\epsilon_{ii} = \frac{V_{REV}}{V_{REV}^0} - 1 = \Delta\Phi$$

2. updating the volumetric frequency $f(r)$

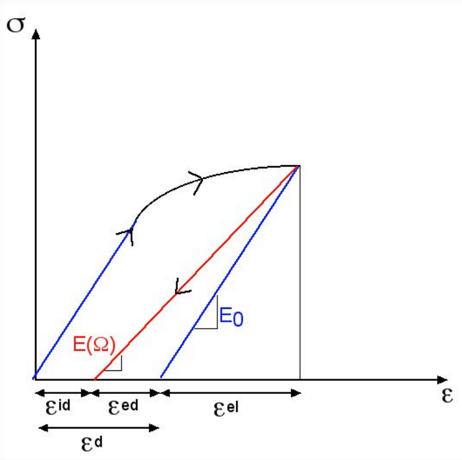
$$f(r) = (\alpha_p(r) + \alpha_c(r))\, \pi\, r^2$$

$\Rightarrow$ **How to update $\alpha_p(r)$ and $\alpha_c(r)$ with the current state variables ?**

$$\int_0^\infty \alpha_p(r)\pi r^2 dr = V_p \qquad \int_0^\infty \alpha_c(r)\pi r^2 dr = V_c$$

$\Rightarrow$ **How to update V_p and V_c with the current state variables ?**

Types of Deformations



$$\Delta V_p = -Tr\left(\epsilon^{\boldsymbol{el}}\right) \qquad \Delta V_c = -Tr\left(\epsilon^{\boldsymbol{d}}\right) \qquad \boldsymbol{D}(\Omega) : \epsilon^{\boldsymbol{id}} = -g\Omega$$

Algorithm (1/2)

1. compute the current increment of damage $d\boldsymbol{\Omega}^{(\boldsymbol{k})}$
2. compute the increment of stress $d\boldsymbol{\sigma}^{(\boldsymbol{k})}$. *For a strain-controlled test :*

$$d\boldsymbol{\sigma}^{(k)} = \boldsymbol{D}\left(\boldsymbol{\Omega}^{(k-1)}\right) : d\epsilon^{(k)} + \left(\frac{\partial \boldsymbol{D}\left(\boldsymbol{\Omega}^{(k-1)}\right)}{\partial \boldsymbol{\Omega}} : \epsilon^{(k-1)}\right) : d\boldsymbol{\Omega}^{(k)} - g d\boldsymbol{\Omega}^{(k)}$$

3. update of the total porous volume $V_v^{(k)}$. *For a strain-controlled test :*

$$\epsilon^{(k)} = \epsilon^{(k-1)} + d\epsilon^{(k)}, \quad {V_v}^{(k)} = -Tr\left(\epsilon^{(k)}\right) + \Phi_0$$

4. update of the volume occupied by cracks $V_c^{(k)}$:

$$d{\epsilon^{el}}^{(k)} = \boldsymbol{D}\left(\boldsymbol{\Omega}^{(k-1)}\right)^{-1} : d\boldsymbol{\sigma}^{(k)}$$

$$d{\epsilon^{d}}^{(k)} = d\epsilon^{(k)} - d{\epsilon^{el}}^{(k)}, \quad {V_c}^{(k)} = -Tr\left({\epsilon^{d}}^{(k)}\right)$$

5. update of the volume occupied by natural pores $V_p^{(k)}$:

$${V_p}^{(k)} = {V_v}^{(k)} - {V_c}^{(k)}$$

Algorithm (2/2)

6. update the mean radius of natural pores $m^{(k)}$:

$$V_p{}^{(k)} = \pi N_p \int_{r^p_{min}}^{r^p_{max}} \left(\frac{1}{s\sqrt{2\pi}} exp\left(-\frac{(r - m^{(k)})^2}{2\, s^2} \right) \right) r^2 dr$$

N_p and s are fixed parameters, determined in the algorithm initialization

7. update the number of cracks present in the REV $N_c^{(k)}$

$$V_c{}^{(k)} = \pi N_c^{(k)} \int_{r^c_{min}}^{r^c_{max}} \frac{1}{\lambda_c} exp\left(-\frac{r}{\lambda_c} \right) r^2 dr$$

λ_c is a fixed parameter, determined in the algorithm initialization

8. update $\alpha_p^{(k)}(r)$ and $\alpha_c^{(k)}(r)$, update the "volumetric frequency" $f^{(k)}(r)$:

$$f^{(k)}(r) = \left(\alpha_p^{(k)}(r) + \alpha_c^{(k)}(r) \right) \pi r^2$$

9. update hydraulic conductivity $k_w{}^{(k)}$:

$$k_w{}^{(k)} = \frac{\gamma}{8\mu} \left(\Phi_0 + Tr\left(-\epsilon^{(k)} \right) \right) \frac{1}{\int_0^\infty f^{(k)}(r) dr} \int_0^\infty f^{(k)}(r) r^2 dr$$

Initialization Process

1. solve the following system of equations for the initial mean radius of the natural pores (m_0), for the standard deviation of the natural pore radius (s, fixed), and for the number of natural pores present in the REV (N_p, fixed) :

$$\begin{cases} V_p^0 = \Phi_0 = \pi N_p \int_{r_{min}^p}^{r_{max}^p} p_p^0(r) r^2 dr \\ m_0 \approx \int_{r_{min}^p}^{r_{max}^p} p_p^0(r) r dr \\ 1 \approx \int_{r_{min}^p}^{r_{max}^p} p_p^0(r) dr \end{cases}$$

2. solve the following equation for the crack characteristic length (λ_c, fixed) :

$$\lambda_c \approx \int_{r_{min}^c}^{r_{max}^c} p_c(r) r dr$$

Summary [Arson and Pereira, 2011]

- 4 input parameters :

$$r^p_{min},\ r^p_{max},\ r^c_{min},\ r^c_{max}$$

- 3 model parameters computed in the algorithm initialization : *mathematical conditions + knowledge of the initial porosity*

$$N_p,\ s,\ \lambda_c$$

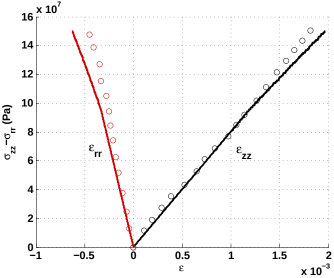
- 2 variables updated with deformation and damage :

$$m,\ N_c$$

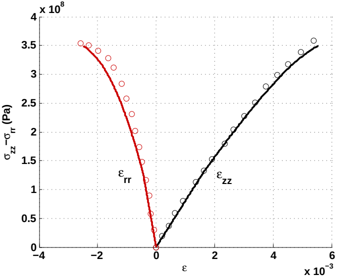
Main Material Parameters (Granite, $\sigma_c = 0$)

E (Pa)	ν (-)	g (Pa)	C_0 (Pa)	C_1 (Pa)	e_0 (-)
$8.01\,10^{10}$	0.28	$-3.3\,10^{8}$	$1.1\,10^{5}$	$2.2\,10^{6}$	0.008
r^p_{min} (μm)	r^p_{max} (μm)	r^c_{min} (μm)	r^c_{max} (μm)		
0.01	1	0.1	10		

Reference Results [Halm and Dragon, 2002 ; Arson and Gatmiri, 2010]

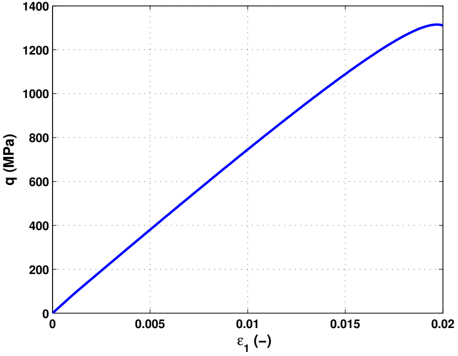


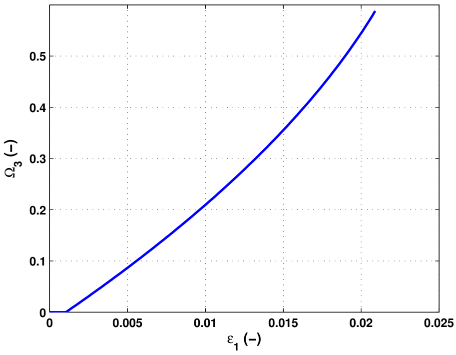
$\sigma_c = 0$ **MPa**



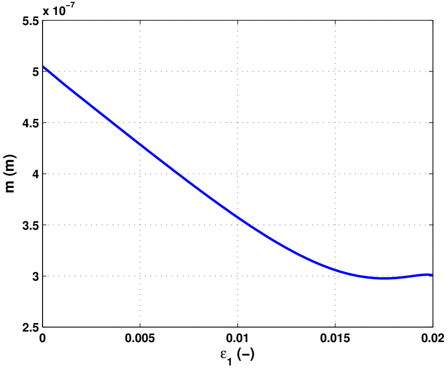
$\sigma_c = 20$ **MPa**

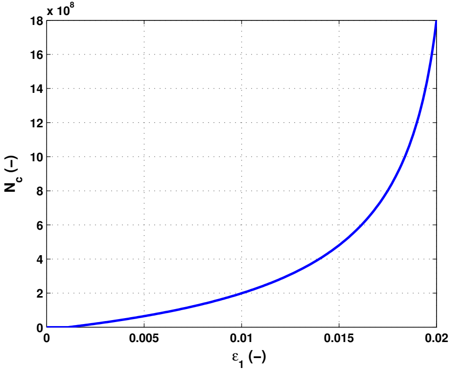
Deviatoric Stress and Damage Evolutions



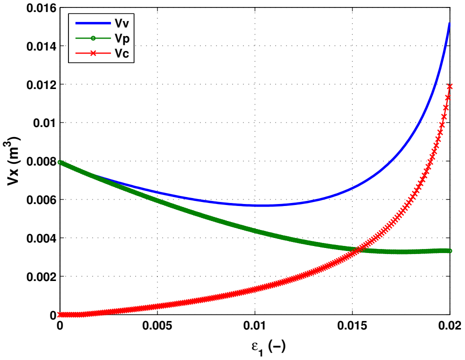


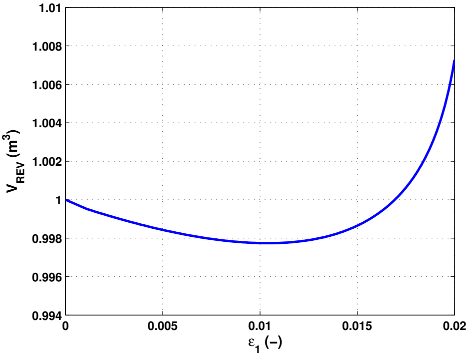
Variations of the Model Variables m and N_c



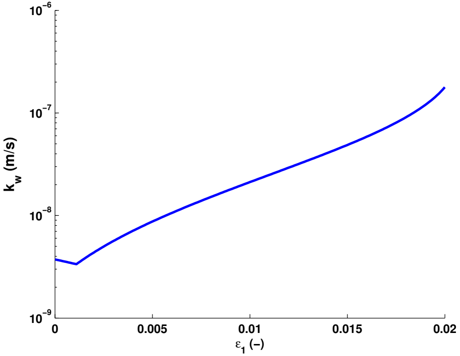


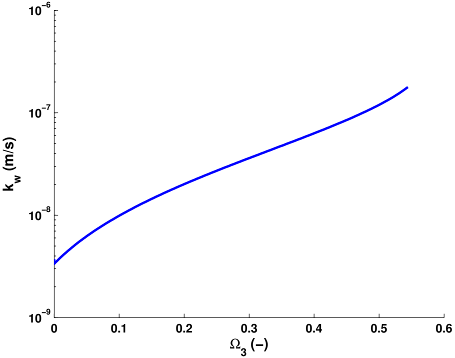
Porous Volume Changes





Impact on Permeability



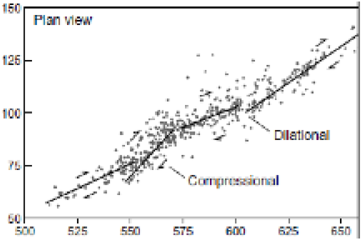


Possible Application : Hydraulic Fracturing

Prospective research : permeability of rock damaged by hydraulic fracturing, from the laboratory scale to the reservoir scale

- crack initiation, propagation, bifurcation, coalescence
- damage due to tectonic stresses (mechanical) and injected fluid pressure (hydraulic)
- crack opening and closure influenced by the presence of a propping agent (sand)

[Busetti et al., 2010]



Research Plans...

- geothermal foundations
- nuclear waste disposals
- oil and gas exploitation by hydraulic fracturing
- high-pressure gas storage
- CO_2 sequestration
- tunneling and mining
- subsidence problems
- mineral exploitation, ore forming mechanisms
- fault processes, diapirs

Research Plans...

Intellectual Merit

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1. **multi-dimensional :** damage anisotropy
   crack interaction, compression unilateral effects, rotation of damage directions

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3. **multi-scale :** microstructure, upscaling method

   growth of pores and defects, connectivity, internal length(s) evolution

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Questions ?